STAT 3011
Spring 2022
Exam 2 (A)
Time Limit: 120 Minutes

Name (Print): SOLUTION

Student ID: ________________________

Instructions:

- Do *not* begin or turn this page until you are instructed.
- Enter all requested information on the top and bottom of this page, and **put your initials on the top of every page**, in case the pages become separated.
- This exam contains 16 pages (including this cover page and the multiple choice answer sheet). Check to see if any pages are missing. There are 14 multiple choice questions and 3 short answer problems.
- The exam is closed book. You may *not* use your books, or any wireless device on this exam.
- You may use a calculator and one sheet of paper (size A4 or 8.5" by 11") with formulas or other notes on both sides. You may *not* share calculators or notes!
- Show all your work on each problem for full credit except multiple choice problems. The following rules apply:
  - *Organize your work*, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
  - *Mysterious or unsupported answers will not receive full credit* for short answer problems. A correct answer, unsupported by calculations, explanation, or algebraic work will not receive full credit; an incorrect answer supported by substantially correct calculations and explanation may still receive partial credit.
  - If you need more space, use the back of the pages; clearly indicate when you have done this.

Honesty Statement and Pledge:

I have not given or received any aid or assistance to or from any other student in this course during the exam period. Everything I have written on this exam represents my own work and knowledge. I sign this knowing that infringements on the University's Academic Honest policy may result in failure or expulsion.

Signed By: ________________________ **Date:** ________________________

Problem 1. (40 points) **Multiple Choice**

Choose the ONLY ONE correct answer for each question. Circle your answers to all questions in the answer sheet provided. (NO explanation is needed).

1. (3 points) Using a random sample of 4000 college students, you construct a 95% confidence interval to estimate the mean sleep hours per day. You decide to compute another 95% confidence interval using a different sample, this time only with 1000 college students. What change would you expect from the first interval to the second?

   (A) The margin of error will be 4 times as large as the first interval.

   *** (B) **The margin of error will be 2 times as large as the first interval.**

   (C) The margin error will be decrease by 75%.

   (D) Standard error will decrease by half.

   margin of error of population mean $= t_{\alpha/2,df}\frac{s}{\sqrt{n}}$

2. (3 points) A 95% confidence interval for the unknown population mean μ is computed from a random sample and found to be 8 ± 2. Which of the following is a correct statement?

   (A) There is a 95% probability that μ is between 6 and 10.

   (B) 95% of values of the population distribution are between 6 and 10.

   *** (C) **If we took many samples and computed a 95% confidence interval repeatedly, approximately 95% of these intervals would contain μ.**

   (D) All of the above are true.

3. (3 points) An instructor at a large university would like to estimate the proportion of students who prefer e-book over physical copy of textbooks. He would like the result to be accurate to within 3% with 90% confidence. How many students should be surveyed? Following R commands may be helpful.

```
qnorm(.9) = 1.28
qnorm(.95) = 1.645
qnorm(.975) = 1.96
pnorm(.9) = 0.82
qt(0.9, df=29) = 1.31
```

   (A) At least 1068

   *** (B) **at least 752**

   (C) At least 456

   (D) At least 187

   $n \geq p^*(1-p^*)\left(\frac{z_{\alpha/2}}{m}\right)^2 = (0.5)(0.5)(1.645/0.03)^2 = 751.6736$

4. (3 points) Suppose that in a certain hypothesis test the null hypothesis is rejected at the $\alpha = .10$ level; it is also rejected at the $\alpha = .05$ level; however it cannot be rejected at the .01 level. The most accurate statement that can be made about the p-value for this test is that:

*** (A) $0.01 <$ **p-value** < 0.05

(B) p-value $= 0.01$

(C) $0.05 < p$-value < 0.10

(D) p-value $= 0.10$

5. (3 points) A Type I error is committed if we make:

(A) a correct decision when the null hypothesis is false.

(B) a correct decision when the null hypothesis is true.

(C) an incorrect decision when the null hypothesis is false.

*** (D) **an incorrect decision when the null hypothesis is true.**

6. (3 points) It is known that for right-handed people, the dominant (right) hand tends to be stronger. For left-handed people who live in a world designed for right-handed people, the same may not be true. To test this, muscle strength was measured on the right and left hands of a random sample of 15 left-handed men and the difference (left - right) was found. The alternative hypothesis is one-sided (left hand stronger). The resulting t-statistic was 1.80. Assuming the conditions are met, based on the t-statistic of 1.80 and the R output

```
> pt(1.8,df=14)
[1] 0.953
> pt(1.8,df=28)
[1] 0.959
```

the appropriate conclusion for this test using $a = .05$ is:

(A) df $= 28$, so p-value $< .05$ and the null hypothesis can be rejected.

*** (B) **df $= 14$, so p-value $< .05$ and the null hypothesis can be rejected.**

(C) df $= 14$, so p-value $> .05$ and the null hypothesis cannot be rejected.

(D) df $= 28$, so p-value $> .05$ and the null hypothesis cannot be rejected.

Alternative hypothesis is $\mu_D > 0$ where D is left-right. Hence P-value is $P(T_{14} > 1.8) = 1 - 0.953 = 0.047$

For Questions 7 and 8: A one-sample t-test for testing $H_0 : \mu = 1.75$ against $H_a : \mu \neq 1.75$ resulted in a p-value of 0.072.

7. (3 points) Based on the same sample, would the 90% confidence interval and the 95% confidence interval for μ contain the value 1.75 ?
   (A) Neither the 90% confidence interval nor the 95% confidence interval would contain 1.75
   (B) Only the 90% confidence interval would contain 1.75
   *** (C) **Only the 95% confidence interval would contain 1.75**
   (D) Both the 90% confidence interval and the 95% confidence interval would contain 1.75

8. (3 points) Based on the same sample and same information as Question 7, can we draw conclusion of a one-sample t-test with $H_0 : \mu = 1.75$ against $H_a : \mu < 1.75$? Assume that the value of test statistic is negative (-).
   (A) No, we don't have enough information
   *** (B) **Yes, we will reject H_0 at significance level 0.05**
   (C) Yes, we will reject H_0 at significance level 0.01
   (D) Yes, we will fail to reject H_0 at significance level 0.05

9. (3 points) Which of the following statements is NOT correct?
   (A) The smaller the p-value, the stronger the evidence that you should reject the null hypothesis.
   *** (B) **If the difference between a point estimate and the hypothesized value is statistically significant, the magnitude of the difference must be large.**
   (C) "Failing to reject" H_0 is not equivalent to "accepting" H_0.
   (D) All of the above are true.

10. (3 points) Suppose that we conduct a hypothesis test with $H_0 : p = 0.4$. Based on a random sample of 40 observations, the sample proportion is $\hat{p} = 0.5$. What is the standard error used to compute the test statistic?
    *** (A) $\sqrt{\frac{0.4(1-0.4)}{40}}$
    (B) $\sqrt{\frac{0.5(1-0.5)}{40}}$
    (C) $\sqrt{0.4(1-0.4)}$
    (D) $\sqrt{0.5(1-0.5)}$

11. (3 points) Suppose in a random sample from UMN students, the average GRE score of engineering major (group **E**) is 320, and the average GRE score of literature major (group **L**) is 310. Which one is the point estimate for the true difference between average GRE scores of engineering major and literature majors? Select the correct statistical notation.

   (A) $\hat{p}_E - \hat{p}_L = 10$
   (B) $p_E - p_L = 10$
*** (C) $\bar{x}_E - \bar{x}_L = 10$
   (D) $\mu_E - \mu_L = 10$

12. (3 points) Select one of the following alternative hypotheses, whose hypothesis test with α significance level is equivalent to the corresponding $1 - \alpha$ confidence interval.

*** (A) $H_a : \mu_1 \neq \mu_2$
   (B) $H_a : \bar{x}_1 \neq \bar{x}_2$
   (C) $H_a : p_1 \neq p_2$
   (D) $H_a : p_1 = p_2$

13. (3 points) In a hypothesis test $H_0 : p_1 = p_2$, $H_a : p_1 > p_2$ with $\alpha = 0.05$, the p-value is 0.00002. Which one of the following is the most possible 95% confidence interval for $p_1 - p_2$?

   (A) (-0.5, 0.5)
*** (B) **(0.5, 0.8)**
   (C) (-0.8, -0.3)
   (D) (1.2, 2.2)

B is correct answer: Since we reject the null and conclude p1>p2, most likely CI is (0.5, 0.8).
Note that D can't be answer because proportions are between 0 and 1.

14. (1 point) Did you circle your multiple choice answers on page 16?

*** (A) **No, but I will now.**
*** (B) **Yes.**
*** (C) **Yes.**
*** (D) **Yes.**

Problem 2. (20 points) Be sure to show all work for full credit.

In this problem we want to estimate the population mean time spent on social media per day for students at the University of Minnesota.

Assume that "survey" data is a random sample of 209 students from the University of Minnesota. We will use the variable named `hours.spent.social.network.per.day`. Below is selected R outputs from data set `survey`.

```
x<-survey$hours.spent.social.network.per.day
mean(x) = 3.3
sd(x) = 2
length(x) = 209
qt(0.9, df=length(x)-1) = 1.29
qt(0.95, df=length(x)-1) = 1.65
qt(0.975, df=length(x)-1) = 1.97
qt(0.99, df=length(x)-1) = 2.34
```

1. (4 points) What is the point estimate of μ: population mean time spent of social media per day for U students? Use correct statistical notation and find its value from R outputs.

$\bar{x} = 3.31$ **(in hours)**

2. (3 points) Based on sample mean and sample standard deviation provided in R command, do you think the population distribution of time spent on social media is normally distributed? Explain.

Given that sample mean is 3.3 and standard deviation is 2, the distribution is not normal. More specifically, skewed to the right.

Copy of R outputs from the previous page

```
x<-survey$hours.spent.social.network.per.day
mean(x) = 3.3
sd(x) = 2
length(x) = 209
qt(0.9, df=length(x)-1) = 1.29
qt(0.95, df=length(x)-1) = 1.65
qt(0.975, df=length(x)-1) = 1.97
qt(0.99, df=length(x)-1) = 2.34
```

3. (4 points) (i) Calculate the standard error of sample mean using R outputs. (ii) Explain what it describes.

Standard error of sample mean $= s/\sqrt{n} = 2.02/\sqrt(209) = 0.1397$
This describes how much sample means vary from sample to sample if we repeatedly select 209 University of Minnesota students randomly and calculate sample mean of time spent on social media.

4. (6 points) (i) From R output above, identify the correct t-multiplier value for a 95% confidence interval for μ. (ii) Use it to construct a 95% confidence interval to estimate the population mean time spent on social media for U students. (iii) Interpret the result in context of the problem.

t-multiplier value: `qt(0.975, df=208)=` **1.97**
95% confidence interval
$\bar{x} \pm t_{0.025,df=208}(s/\sqrt{n}$
$= 3.3 + (1.97)(2/\sqrt{209})$
$= 3.3 \pm 0.273$
$= (3.027, 3.573)$
We are 95% confident that the population mean time spent on social media for the University of Minnesota students is between 3.027 and 3.573.
OR
If we select a random sample of 209 students and construct a 95% confidence interval for μ repeatedly, then 95% of these intervals would contain the true population mean time spent on social media.

5. (3 points) Based on the interval from 4, is it plausible that students spend 4 hours on social media on average? Explain.

No, it is not plausible because 4 is not contained in the 95% confidence interval.

Problem 3. (20 points) Be sure to show all work for full credit.

It is known that males study around 18.5 hours/week. You conducted a survey of 272 randomly selected females attending the University of Minnesota and asked them how much they study per week on average. You calculated the following summary statistics: $\bar{x} = 22.68$, $s = 11.83$. Let μ be the true mean hours studying per week by females. Below are some R outputs that might be helpful.

```
pt(5.83,df=271,lower=F) = 7.860964e-09
pt(5.83,df=271) =  1
pt(5.83,df=272,lower=F) = 7.833126e-09
pt(5.83,df=272) = 1
qt(0.05,df=271) = -1.650496
qt(0.025,df=271) = -1.968756
qt(0.05,df=272) = -1.650475
qt(0.025,df=272) = -1.968724
```

1. (8 points) Statistically test whether females differ from males in the average amount of studying per week. Perform all 5 steps of hypothesis testing at $\alpha = 0.05$.
   For the p-value, circle the exact line of R code you used from above.

1. Assumptions: We assume sample is randomly selected. Sample size is larger than 30, so we can assume average time studying is normally distributed.

2. Hypotheses: $H_0 : \mu = 18.5$ **vs.** $H_a : \mu \neq 18.5$.
3. Test statistic: $T = \frac{22.68-18.5}{11.83/\sqrt{272}} = 5.83$.
4. p-value: $2P(t > |5.83|) = 2 * pt(5.83, df = 271, lower = F) = 1.572193e-08 \approx 0$.
5. Conclusion: Because the p-value is much less than 0.05, we reject Ho and conclude that there is enough evidence to indicate that the average amount of time females spend is different from males.

Copy of R outputs from Problem 3 description.

```
pt(5.83,df=271,lower=F) = 7.860964e-09
pt(5.83,df=271) =  1
pt(5.83,df=272,lower=F) = 7.833126e-09
pt(5.83,df=272) = 1
qt(0.05,df=271) = -1.650496
qt(0.025,df=271) = -1.968756
qt(0.05,df=272) = -1.650475
qt(0.025,df=272) = -1.968724
```

2. (1 point) What is the p-value if you wanted to test whether the number of hours females study per week is greater than males at $\alpha = 0.05$? **For the p-value, circle the exact line of R code you used from above.**

P-value for a one-sided test is half of a two-sided test or pt(5.83, df=271, lower=F) = 7.860964e-09 ≈ 0.

3. (3 points) Formulate, calculate, *and interpret* a 95% confidence interval for μ.

$\bar{x} \pm t_{\alpha/2,n} \frac{s}{\sqrt{n}} = 22.68 \pm t_{0.025,271} \frac{11.83}{\sqrt{271}} = 22.68 \pm 1.4148 = (21.27, 24.09)$**, where -qt(0.025, df=271)=1.968756. We are 95% confident that the true mean hours of studying per week spent by males is between 21.27 and 24.09.**

4. (2 points) Does the interpretation of the confidence interval from Question 3 agree or disagree with the results from the statistical test in Question 1? Explain your decision statistically.

Yes, it agrees. Since 18.50 is not contained within the 95% confidence interval, it agrees with our conclusion to reject H_0 at $\alpha = 0.05$.

5. (2 points) State the Type I error in the context of this problem. What is the probability of this Type I error based on Question 1?

We conclude that the amount of time females spend studying is different from males when in fact there is no difference. The probability is 0.05.

6. (2 points) State the Type II error in the context of this problem.

We conclude that there is not difference in the amount of time males and females spend studying when in fact there is a difference.

7. (2 points) Which type of error (Type I or Type II) could you have made in Question 1? Explain your choice.

Since we rejected Ho, we could have made a Type I error.

Problem 4. (20 points) Be sure to show all work for full credit.

The researchers in a pharmaceutical company want to test the effectiveness of a new medicine towards certain disease. They take a sample of 100 patients and **divided them equally into two groups**: Treatment group and Control group. The patients in the treatment group are treated with the new medicine and the patients in the control group are given the traditional treatment. Let p_1 denote the true effective rate of the new medicine and p_2 denote the true effective rate of the traditional treatment. The true difference between two rates: $p_1 - p_2$ is called the **risk difference**.

Suppose 27 patients in the treatment group and 11 patients in the control group have shown curative effect (the treatment is working well) at the end of the study.

You may find the following R commands useful.

```
qnorm(0.98) = 2.05
qnorm(0.99) = 2.33
```

1. (2 points) Find a point estimate for the risk difference, $p_1 - p_2$. Use the correct notation and find the value of the point estimate.

1) $\hat{p}_1 - \hat{p}_2 = 27/50 - 11/50 = 16/50 = 0.32$

2. (4 points) Construct and interpret a 98% confidence interval for the risk difference.

2)

$$\hat{p}_1 - \hat{p}_2 \pm z_{0.99}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$
$$=0.32 \pm 2.33 \times \sqrt{0.54 \times 0.46/50 + 0.22 \times 0.78/50}$$
$$=0.32 \pm 2.33 * 0.092 = (0.106,\ 0.534)$$

We are 98% confident the the true risk difference is between 0.106 and 0.534. (or equivalent interpretations)

Copy of Problem 4 description and R outputs from the previous page.

The researchers in a pharmaceutical company want to test the effectiveness of a new medicine towards certain disease. They take a sample of 100 patients and **divided them equally into two groups**: Treatment group and Control group. The patients in the treatment group are treated with the new medicine and the patients in the control group are given the traditional treatment. Let p_1 denote the true effective rate of the new medicine and p_2 denote the true effective rate of the traditional treatment. The true difference between two rates: $p_1 - p_2$ is called the **risk difference**. Suppose 27 patients in the treatment group and 11 patients in the control group have shown curative effect (the treatment is working well) at the end of the study.

You may find the following R commands useful.

```
qnorm(0.98) = 2.05
qnorm(0.99) = 2.33
```

3. (4 points) The researchers want to test whether the new medicine works better than the traditional treatment. i) Check the assumptions of the test and ii) state the alternative hypothesis.

3) Assumptions: 2 random independent samples, at least 5 successes and failures in each group. $H_a : p_1 - p_2 > 0$ **or** $p_1 > p_2$**.**

4. (5 points) i) Write down the formula of test statistic and ii) its distribution under null hypothesis. iii) Calculate the value of the test statistic **with 1 decimal place**.

4)

$$
\begin{aligned}
\hat{p} &= (27 + 11)/100 = 0.38 \\
z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}} \sim N(0,1) \\
&= 0.32/\sqrt{0.38 \times 0.62(1/50 + 1/50)} \\
&= 3.3
\end{aligned}
$$

5. (2 points) Recall the 68-95-99.7 rule for Normal distribution, probability of Normal distribution within 1, 2, and 3 standard deviations away from the mean is approximately 0.68, 0.95, and 0.997 respectively.
Based on the test statistic from question 4 on page 13, state the conclusion of this hypothesis test at level $\alpha = 0.0025$ without calculating exact p-value.

P-value = $P(Z > 3.3)$**, by 68-95-99.7 rule, the probability** $P(-3 < Z < 3) = 0.997$**, so** $P(Z > 3.3) < P(Z > 3) = 0.0015 < 0.0025$**, so reject null hypothesis.**

Do not use any information above.

6. (3 points) Suppose that researchers want to compare the risk rates of two brands of medicine (brand A v.s. brand B) while maintaining the margin of error at most 0.1 for a 98% confidence interval for $p_A - p_B$ where p_A is the true effective rate of brand A and p_B is that of brand B.
To minimize the expenses, the researchers want to use as few patients as possible. What is the smallest sample size needed?
Use $\hat{p}_A = 0.54$ and $\hat{p}_B = 0.45$ as educated estimates of p_A and p_B.
Remember n must be even so that the n patients can be divided equally into two groups.
You may find the following R commands useful.

```
qnorm(0.98) = 2.05
qnorm(0.99) = 2.33
```

$n_A = n_B = n/2$

$$\begin{aligned} moe &= z_{0.99}\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n_A} + \frac{\hat{p}_B(1-\hat{p}_B)}{n_B}} \\ &= 2.33 \times \sqrt{0.54 \times 0.46/(n/2) + 0.45 \times 0.55/(n/2)} \\ &= 2.33 \times \sqrt{0.9918/n} \leq 0.1 \\ n &\geq \left(\frac{2.33 \times \sqrt{0.9918}}{0.1}\right)^2 = 538.44 \end{aligned}$$

So n **is at least 540 (smallest even integer> 538.44).**

Name: ________________________

Lecture Section:	001	006	011	016
Lecture time:	11:15 am	8:00 am	12:20 pm	10:10 am
(**Circle One**)	Park	Yang	Xu	Shen

Question	Answer			
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D

Please do NOT write in the following table. This is for grading purpose only!

Question	1	2	3	4	100
Score					
Total	40	20	20	20	100