STAT3011
Fall 2021
Exam 2 (B)
Time Limit: 120 Minutes

Name (Print): SOLUTION

Student ID: ____________________

Instructions:

- Do *not* begin or turn this page until you are instructed.
- Enter all requested information on the top and bottom of this page, and put your initials on the top of every page, in case the pages become separated.
- This exam contains 14 pages (including this cover page and the multiple choice answer sheet). Check to see if any pages are missing. There are 1 multiple choice problem (with 14 questions) and 3 short answer problems.
- The exam is closed book. **Do not** use your books, notes, or any wireless device on this exam.
- You may use a calculator and one sheet of paper (size A4 or 8.5" by 11") with formulas or other notes on both sides. You may *not* share calculators or notes!
- Show all your work on each problem for full credit except multiple choice problems. The following rules apply:
  - *Organize your work*, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
  - *Mysterious or unsupported answers will not receive full credit* for short answer problems. A correct answer, unsupported by calculations, explanation, or algebraic work will not receive full credit; an incorrect answer supported by substantially correct calculations and explanation may still receive partial credit.
  - If you need more space, use the back of the pages; clearly indicate when you have done this.

Honesty Statement and Pledge:

I have not given or received any aid or assistance to or from any other student in this course during the exam period. Everything I have written on this exam represents my own work and knowledge. I sign this knowing that infringements on the University's Academic Honest policy may result in failure or expulsion.

Signed By: ____________________ **Date:** ____________________

Problem 1. (37 points) **Multiple Choice**

Choose the ONLY ONE correct answer for each question. Circle your answers to all questions in the answer sheet provided. (NO explanation is needed).

1. (3 points) Based on 1000 respondents, a 95% confidence interval for the mean number of close friends equals $(6.8, 8.0)$. Which of the following interpretation is correct?
   (i) Ninety-five percent of the values of X = number of close friends are between 6.8 and 8.0.
   (ii) If random samples of 1000 people were repeatedly selected, then 95% of the time, confidence interval would contain the population mean, μ.
   (A) Both (i) and (ii) are correct
   (B) (i) is correct and (ii) is incorrect
   *** (C) **(i) is incorrect and (ii) is correct**
   (D) Both (i) and (ii) are incorrect

2. (3 points) Which of the following will cause the width of the confidence interval for p to increase, assuming everything else remains unchanged? Consider each statement separately.
   (i) Increase the confidence level.
   (ii) Increase $\hat{p}$ from 0.7 to 0.9.
   (iii) Increase sample size.
   *** (A) **(i)**
   (B) (i) and (ii)
   (C) (ii)
   (D) (ii) and (iii)

Width of CI is twice margin of error, $2z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p}}{n}}$
ii) When $\hat{p} = 0.7$**, moe is** $z_{\alpha/2}\sqrt{0.7(0.3)/n} = z_{\alpha/2}\sqrt{0.21/n}$
When $\hat{p} = 0.9$**, moe is** $z_{\alpha/2}\sqrt{0.9(0.1)/n} = z_{\alpha/2}\sqrt{0.09/n}$
Hence the width decreases.

3. (3 points) Suppose you want to calculate a 99% confidence interval for women's average height, using the data of 30 randomly selected women. What should be the value of the multiplier of the confidence interval?
   (A) `qt(0.99, df = 30)`
   *** (B) `qt(0.995, df = 29)`
   (C) `qt(0.01, df = 30)`
   (D) `qnorm(0.005)`

4. (3 points) Suppose we want to know how many observations needed to construct a confidence interval for the average UMN students' height with a reasonably small margin of error. Which of the following will cause the sample size needed to increase, assuming all other factors remain the same? Consider each statement separately.

   (A) The total number of students at UMN increases.

   (B) The desired margin of error gets larger.

*** (C) **The confidence level increases.**

   (D) None of above.

$n \geq \left(\frac{z_{\alpha/2}s^*}{m}\right)^2$ **where m is the desired margin of error.**
As the confidence level increases, $z_{\alpha/2}$ increases.

5. (3 points) Which of the following is NOT a correct way to state a null hypothesis?

   (A) $H_0 : p = 0.5$

*** (B) $H_0 : \hat{p}_1 = \hat{p}_2$

   (C) $H_0 : \mu = 10$

   (D) $H_0 : \mu_1 = \mu_2$

6. (3 points) Which of the following is true about the standard error of a statistic?

*** (A) **Different form of standard error of sample proportion is used in hypothesis test for proportion and confidence interval for proportion.**

   (B) Different form of standard error of sample mean is used in hypothesis test for mean and confidence interval for mean.

   (C) Standard error increases as sample size(s) increases.

   (D) All of the above are true.

Standard error of sample proportion for hypothesis testing is $\sqrt{\frac{p_0(1-p_0)}{n}}$ and for confidence interval is $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

7. (3 points) Academic committee claims the average time that students spend on studying is greater than 40 hours/week. Suppose they took a random sample of 100 students and found the sample mean hours was 41.5 with a sample standard deviation of 6. What is the test statistic for this test?

   (A) -0.25

(B) 0.25

(C) -2.5

*** (D) **2.5**

8. (3 points) In tests of a computer component, it is found that the mean time between failures is 520 hours. A modification is made which is supposed to increase the time between failures. Assume that the time between failures are approximately normally distributed. A hypothesis test was conducted to see if the modification was effective. Use the following R output.

```
One Sample t-test
 data:  x
 t = 2.8204, df = 9, p-value = 0.01002
 alternative hypothesis: true mean is greater than 520
 95 percent confidence interval:
 526.4411      Inf
sample estimates:
mean of x
    538.4
```

Select the most appropriate statement: The p-value=0.01002 indicates that

(A) The probability of making a Type I error is 0.01002.

(B) The probability that $\mu = 520$ is 0.01002.

*** (C) **There is strong evidence against μ= 520 since if μ were equal to 520, it would be unusual to obtain a value as or more extreme than $\bar{x}$ = 538.4**

(D) There is strong evidence that $\mu = 520$ since if μ were greater than 520, the probability that $\bar{x} = 538.4$ is only 0.01002.

9. (3 points) What is the point estimation for $\mu_1 - \mu_2$?

(A) $\mu_1 - \mu_2$

*** (B) $\bar{x}_1 - \bar{x}_2$

(C) μ_D

(D) $\hat{p}_1 - \hat{p}_2$

10. (3 points) A two-sample independent mean test for testing $H_0 : \mu_1 = \mu_2$ against $H_a : \mu_1 \neq \mu_2$ resulted in a p-value of 0.08. Based on this result, would the 95% confidence interval and the 99% confidence interval for $\mu_1 - \mu_2$ contain the value 0?

(A) Neither the 95% confidence interval nor the 99% confidence interval would contain 0

*** (B) **Both the 95% confidence interval and the 99% confidence interval would contain 0**

(C) Only the 95% confidence interval would contain 0

(D) Only the 99% confidence interval would contain 0

If $\alpha = 0.05$ and P-value 0.08 > 0.05, we fail to reject the null hypothesis $H_0 : \mu_1 = \mu_2$. A 95% confidence interval results the same conclusion; it is plausible that $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$. Hence 95% CI will contain 0. The same goes for $\alpha = 0.01$ and 99% confidence interval.

11. (3 points) One instructor wants to test whether the exam scores of the two lab sections are significantly different. Which hypothesis test should this instructor use?

    (A) One sample t-test

    *** (B) **Independent two-sample t-test**

    (C) Matched-pairs t-test

    (D) One sample z-test

12. (3 points) Suppose we are interested in the protein concentration (in grams/kg of wheat) of a winter wheat and a spring wheat. The growth and protein level of these wheats may be influenced by the location in which they're grown (temperature, altitude, soil characteristics, etc). Therefore, we select 10 different locations and plant both a winter wheat and a spring wheat in each location. That is, winter and spring wheat observations are matched by location. The following is a partial data set for our experiment:

Location	Spring Wheat Protein	Winter Wheat Protein	Difference (spring − winter)
1	122	87	35
2	171	145	26
3	144	116	28
⋮	⋮	⋮	⋮

What is the number of degrees of freedom associated with the appropriate t-test for testing to see if there is a difference between the mean number of lesions per leaf produced by the two strains?

    (A) 8

    *** (B) **9**

    (C) 10

    (D) 11

13. (1 point) Did you circle your multiple choice answers on page 14?

    *** (A) **No, but I will now.**

    *** (B) **Yes.**

    *** (C) **Yes.**

    *** (D) **Yes.**

Problem 2. (20 points) Be sure to show all work for full credit.

John thinks the average height of male UMN students is 6 feet. He randomly selected 40 male UMN students and found the sample mean is 5.8 feet and the sample standard deviation is 0.1 in feet.

The following R outputs may be helpful.

```
qt(0.975, df = 39) = 2.023
qt(0.95, df = 39) = 1.68
pt(0.975, df = 39) = 0.832
pt(0.95, df = 39) = 0.826
```

1. (8 points) Construct a 95% confidence interval for UMN male students' mean height using John's data. Remember to check the assumptions first.

Since the sample size is 40, and this is a random sample, the assumptions for the CI are satisfied. By the formula, the 95% CI for is

$$\begin{aligned}&\bar{x} \pm t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\\ =&5.8 \pm t_{0.025,39}\frac{0.1}{\sqrt{40}}\\ =&5.8 \pm 2.023 \times 0.016\\ &(5.768, 5.832)\end{aligned}$$

2. (6 points) Interpret the confidence interval from the previous question. Do you think John over- or underestimated the mean height? Why?

We are 95% confident that the true mean height is between 5.768 feet and 5.832 feet. Because 6 is not in this interval, it is likely that John has overestimated the mean height.

3. (6 points) John thinks the confidence interval is too wide and he wants to collect more data to shorten the width of the confidence interval down to 1/3 of the current one. What is the smallest sample size to achieve John's goal?
Use the sample standard deviation 0.1 as an educated estimation s^*.

In addition the R outputs above, the following may also be helpful.

```
qnorm(0.975) = 1.96
qnorm(0.95) = 1.64
pnorm(0.975) = 0.84
pnorm(0.95) = 0.83
```

Based on John's study, we know that the true standard deviation is around 0.1**. Thus by the formula, the desired sample size is**

$$\left(\frac{z_{\alpha/2} \times 0.1}{0.032/3}\right)^2 = \left(\frac{1.96 \times 0.1}{0.032/3}\right) = 337.64 \approx 338.$$

Problem 3. (20 points) Be sure to show all work for full credit.

For Problems 1 - 3:
A survey was conducted to investigate the overall satisfaction rate and the true weekly work hours among randomly chosen 80 employees in a company. The Human Resources department claims that more than 80% of the employees are satisfied with their current job. 66 of the chosen employees said "Yes". Consider a hypothesis testing to prove the claim by the HR department is correct at level 0.05.

```
> pnorm(-0.559)
[1] 0.288
```

1. (4 points) State the null and alternative hypotheses in terms of p, the proportion of employees that are satisfied with their current job.

$$H_0 : p = 0.8$$
$$H_a : p > 0.8$$

2. (4 points) Can we use the large-sample hypothesis testing for proportion in this case? Check the large sample size assumptions.

Yes. Since $80 \times 0.8 = 64 > 15$ **and** $80 \times 0.2 = 16 > 15$**, the assumption is valid.**

3. (7 points) Compute the test statistic and find the p-value. Which type of error could you have made? State the error in the context of this question.

$$z = \frac{\frac{66}{80} - 0.8}{\sqrt{\frac{0.8*0.2}{80}}} = \frac{0.825 - 0.8}{\sqrt{0.002}} = 0.559$$

The corresponding p-value is $P(Z > 0.559) = P(Z < -0.559) = 0.288$. Therefore, we fail to reject the null hypothesis. Type II error is only one type of error we could have made in this case. That is, we fail to reject that the population proportion is 0.8 while the truth is that it is greater that 0.8.

4. (5 points) Let μ be the population mean of the true weekly work hours. Now suppose that we want to test $H_0 : \mu = 40$ vs. $H_a : \mu > 40$ at level 0.05 and that a 90% confidence interval based on the sample doesn't cover 40 and its lower bound is greater than 40. Could we reject the null hypothesis? Prove your answer with inequalities such as p-value < 0.05, p-value > 0.05, etc.

Suppose that the observed t-score is t. Since the lower bound of CI is greater than 40, and by the equivalence between two-sided hypothesis testing and confidence interval, we immediately have that

$$2P(T_{79} > |t|) = 2P(T_{79} > t) < 0.1.$$

Therefore, we have

$$P(T_{79} > t) < 0.05,$$

which means the p-value for the desired test is smaller than 0.05 and hence we reject the null hypothesis.

Problem 4. (20 points) Be sure to show all work for full credit.

A large automobile manufacturing company is trying to decide whether to purchase brand A or brand B tires for its new models. To help arrive at a decision, an experiment is conduct using randomly selected 12 of each brand. The tires are run until they wear out and the distance each tire ran was recorded in kilometers. Test the hypothesis that the brand A tire's average wear is different from brand B tires' with $\alpha = 0.05$. Assume the populations to be approximately normally distributed. Following R code may or may not be helpful.

```
> brandA<-c(36720.48, 30245.67, ....)
> brandB<-c(33132.05, 42034.55, ....)
> mean(brandA) = 35841
> mean(brandB) = 37449
> sd(brandA) = 5075
> sd(brandB) = 6884
> sd(brandA-brandB) = 10381
```

1. (6 points) Specify the assumptions and the hypotheses.

Assumption: Two independent random samples, both popoulations are normally distributed.
$H_0 : \mu_1 = \mu_2$
$H_a : \mu_1 \neq \mu_2$ **where group 1 is Brand A and group 2 is Brand B.**

2. (6 points) Calculate the test statistic.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}} = \frac{35841 - 37449}{\sqrt{\frac{5075^2}{12} + \frac{6884^2}{12}}} = -0.651$$

3. (5 points) Report the p-value. Following R code may or may not be helpful. Interpret the p-value.

```
> pt(-0.536, df=11) = 0.301
> pt(-0.651, df=11) = 0.264
> pnorm(-0.536)= 0.295
> pnorm(-0.651) = 0.257
```

p-value = 2*`pt(-0.651, df=11)` = 2*0.264 = 0.528.
If there was no difference in average wear's brand A vs brand B, then the probability of obtaining test statistic -0.651 or more extreme is 0.528.
This is not unlikely if the null is true.

4. (3 points) **Multiple choice question**: Select the most correct conclusion.

   (a) There is strong evidence to conclude that two brands have the same average wear.
   (b) There is no strong evidence to conclude that two brands have different average wears.
   (c) There is no strong evidence to conclude that two brands have the same average wear.
   (d) None of the above.

(b)

Name: ____________________________

Lecture Section:	001	005	009	013	017
Lecture time:	9:05 am	8:00 am	12:20 pm	10:10 am	11:15 am
(**Circle One**)	Zhao	Shen	Xu	Park	Song

Question	Answer			
1	A	B	C	D
2	A	B	C	D
3	A	B	C	D
4	A	B	C	D
5	A	B	C	D
6	A	B	C	D
7	A	B	C	D
8	A	B	C	D
9	A	B	C	D
10	A	B	C	D
11	A	B	C	D
12	A	B	C	D
13	A	B	C	D
14	A	B	C	D

Please do NOT write in the following table. This is for grading purpose only!

Question	I	II	III	IV	Total
Score					
Out of	40	20	20	20	100